

Range Validation Using Kalman Filter Techniques

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A method is presented whereby range pseudo-residuals may be improved to the level required for the purposes of data validation. Basically, the method proposed is a measurement updating process which utilizes Bierman's adaptation of the Kalman filter measurement updating algorithms together with process noise compensation to account for model errors. This algorithm involves combining the currently available range predictions and measurements to produce an updated range residual measurement whose accuracy is constrained by the range data quality and by the estimated error in the prediction. The algorithm is compact and fast, and is thus suitable for on-line applications in network control or at the station.

I. Introduction

The problem of near-real-time validation of ranging data has, to now, never been completely resolved mainly because the output of the ranging machine (Planetary Ranging Assembly, hereafter referred to as the PRA) is only a vernier measurement and not an absolute measure of distance or elapsed time. The problem is well described by Berman (Ref. 1) in an article which describes a technique for validating range within a pass by using the range and doppler data types to produce a "pseudo-DRVID" which can be used for this purpose. This approach proved fruitful for validating data within a pass but left the problem of validating range data after intervals when no radio metric data has been collected unresolved.

In general the problem occurs because the predicted observables are dependent on the probe ephemeris determined by the Orbit Determination Program (ODP) on the basis of fitting a

model to past observations. Errors in the determination of this ephemeris show up as an error bias and an error drift rate in the predicted range observables. When these predictions are beat against the actual observables in near-realtime, the error growth eventually causes the residuals¹ to separate beyond the limits of the modulo and, in fact, if the data are sparse, there may be several modulus lost in the interval. Such conditions render the residuals almost useless for the purposes of data validation.

There are three basic ways by which this problem can be resolved:

- (1) Place requirements on the flight project to maintain the predicted ephemeris at a specified level of accuracy.

¹ These residuals are usually termed pseudo-residuals to distinguish them from the actual ODP residuals.

- (2) Develop a special purpose differential correction computer program for network control operations which would correct the orbital elements based on the incoming data so as to reduce the drift and bias errors in the predicted ephemeris.
- (3) Develop statistical and/or numerical techniques using only the data types and information currently available to the DSN to compute improved range estimates from the predicted ephemeris.

The first method, although not requiring new software, would be costly and is contrary to the present mode of operation. The reason for this is that the Navigation Team updates the orbit in terms of weeks of data, and orbit predicts are produced for 8 days at a time based on orbits that may be updated once a month. To change this method of operation to meet the requirements would mean tracking and updating the probe ephemeris almost every other day. Considering the workload that current operations are placing on an already overburdened system, one must reject this first approach as being unrealistic.

The second approach, though seemingly reasonable, nevertheless has its own drawbacks. A differential correction process to produce dynamically corrected orbital elements would result in a sizeable computer program which, because of the demand to be made upon it, would require that either a mini- or a midi-computer be dedicated to this purpose alone. Furthermore, such a computer program could not be operated with untrained personnel; a team of knowledgeable technicians would be required. There is currently no in-house expertise of this type within the DSN. Even if these problems were to be resolved, the cost and time of developing and checking out such a program could be considerable.

The third approach appears capable of performing the task within reasonable cost and effort bounds. Berman (Refs. 1 and 2) has achieved partial success by employing doppler data to validate range measurement within a tracking pass or within contiguous passes. The method presented here demonstrates how currently available range predictions and measurements can be sequentially processed in an on-line fashion, using a Kalman filter (Ref. 3) to permit validation of range even when there are substantial gaps in the data coverage. By modeling the errors on the predicted observables, the method has the potential of being able to maintain residuals of sufficient accuracy to detect system malfunctions for periods of up to a year with data taken every 20 days.

Previous numerical experiments documented in Ref. 4 have demonstrated the efficiency, stability, and reliability of the U-D factorized Kalman filter (cf Refs. 3 and 5). A U-D filter

formulation was chosen because the ultimate goal is to develop a real-time algorithm that can function on a limited precision minicomputer.

The filter model described in Section III was tested with simulated data and with real data obtained from one of the Viking spacecraft. A simple linear stochastic filter was developed and applied to the Viking data, giving accurate range residual estimates for the four-month period of data available for our test. Although further tests and analysis remain to be performed, the initial results reported demonstrate the efficiency of the approach.

II. Problem Formulation

The major cause for divergence of the predicted spacecraft range is the effect of unmodeled velocity and acceleration errors. The basis of the approach is to assume that the systematic components of these errors along the line-of-sight from the earth to the spacecraft can be modeled locally with a low order polynomial. This assumption is consistent with past analyses of spacecraft tracking errors (Ref. 6), and is evidenced in the characteristics of our test data (Section IV). Figure 1 illustrates the typical localized behavior of this error process when the spacecraft trajectory is not affected by any gravitational forces other than those due to the sun.

The actual signal traversal time between the observing station and the spacecraft is designated T_a , while the traversal time predicted from the spacecraft ephemeris is designated T_e . The time difference measured as an observable, T_M , may then be represented as

$$T_M = T_a + \nu(t) - n(t)M \quad (1)$$

where

$\nu(t)$ = noise on the data including errors due to medium effects

M = modulo as determined by frequency and hardware characteristics

$n(t)$ = integral number of modulus inherent in the round-trip delay

$$= [(T_a + \nu(t))/M]_{\text{int}} \text{ (see footnote 2)}$$

² $[a]_{\text{int}}$ indicates the integral part of a .

A residual is produced by differencing the observed measurement from a nominal estimate obtained from the spacecraft ephemeris. Thus,

$$\Delta T_M = T_M - \hat{T}_M \quad (2)$$

where

$$\hat{T}_M = T_e - \hat{n}(t)M$$

$$\hat{n} = \text{integral number of modulus that enter into } T_e, \\ \text{i.e., } \hat{n} = [T_e/M]_{\text{int}}$$

Assume that the difference between the actual and observed measurements may be expressed as

$$\Delta T_e = T_a - T_e \cong \epsilon_0(t) + \dot{\epsilon}_0 t + \frac{1}{2} \ddot{\epsilon}_0 t^2 \quad (3)$$

Then the observable difference, ΔT_M , may be written as

$$z(t) = [a^T(\Delta t_j) \epsilon(t_j) + \nu(t)] \text{ modulo } M \quad (4a)$$

or

$$z(t) = a^T(\Delta t_j) \epsilon(t_j) + \nu(t) - \Delta n(t)M \quad (4b)$$

where

$$z(t) = \Delta T_M$$

$$\Delta n(t) = n(t) - \hat{n}(t)$$

$$a^T(\Delta t_j) = (1, \Delta t_j, (\Delta t_j)^2/2)$$

$$\Delta t_j = t - t_j$$

$$\epsilon^T(t_j) = (\epsilon(t_j), \dot{\epsilon}(t_j), \ddot{\epsilon}(t_j))$$

The role of t_j is to periodically introduce a time shift so that the expansion, Eq. (3), can better retain its validity. Mapping equations corresponding to this time update are

$$\epsilon(t_j) = \Phi(t_j) \epsilon(t_{j-1}) + B\omega_j \quad (5)$$

where

$$\epsilon(t_j) = \text{3-component state vector at time } t_j$$

$$\Phi(t_j) = \text{state transition matrix from time } t_{j-1} \text{ to } t_j$$

$$\omega_j = \text{stochastic white noise}$$

$$B = [0, 0, 1]^T$$

With the exception of the modulo M term in Eq. (4), the problem described is an ordinary parameter estimation or filtering problem (depending on whether the acceleration time constant, τ , of Eq. (7) is small or large). The structure of the Kalman filter is well suited to this type of problem. Its recursive structure allows one to perform the following functions:

- (1) Recursively estimate (t_j) and produce an estimate of the innovations variance, $E[z(t_j) - a^T(\Delta t_j) \epsilon(t_j|t_{j-1})]^2$ for use in data validation.
- (2) Detect and pass outlier points; viz., points having an observation error greater than, say, 3σ , are flagged and omitted from the filtering process (σ is the innovations standard deviation).
- (3) Maintain the condition $|z(t)| < M$, i.e., keep \hat{T}_M close to T_M .

Maintaining the modulo constraint is of principal importance because it allows us to deal with an essentially linear problem. This can be seen clearly by referring to Eq. (4b) and noting that the differences between the observation z_j and the filter predicted estimate $\tilde{z}_j = z(t_j|t_{j-1})$ is given by

$$\begin{aligned} \Delta z_j &= z_j - \tilde{z}_j \\ &= a^T(\Delta t_j) (\epsilon(t_j) - \epsilon(t_j|t_{j-1})) + \nu(t_j) - \Delta n(t)M \end{aligned} \quad (6)$$

where

$$\Delta n(t) = n - \hat{n} = [(\Delta T_e + \nu)/M]_{\text{int}}$$

The validity of the rejection criterion depends on the assumption that $\Delta n = 0$. As long as this is true, the data can be processed in a conventional manner. Whenever $|\Delta z| > 3$, an additional test is made to determine whether $\Delta n \neq 0$. If it is, the datum can be corrected for the modulo rollover and the

standard rejection criterion can then be applied. The test is based on the assumption that

$$|\Delta z_j| \ll M/2$$

When $\Delta t > 0$, either our estimated function or the actual data must go through modulo rollovers at distinct times. When this happens, the anomaly can be detected because, then,

$$|\Delta z_j| > |M - 3\sigma|$$

Note how this rollover procedure introduces a major non-linearity into the problem structure.

III. A Compact and Efficient Recursive Filter Solution

In this section a recursive filter algorithm is described that is well suited for on-line range calibration. Besides being compact and efficient, the mechanization (Refs. 3 and 5) is numerically accurate and stable. The specific form chosen for our dynamic model is

$$\begin{bmatrix} \epsilon \\ \dot{\epsilon} \\ \ddot{\epsilon} \end{bmatrix}_{j+1} = \begin{bmatrix} 1 & \Delta t & \Delta^2 t/2 \\ 0 & 1 & \Delta t \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \epsilon \\ \dot{\epsilon} \\ \ddot{\epsilon} \end{bmatrix}_j + \begin{bmatrix} 0 \\ 0 \\ w_j \end{bmatrix} \quad (7)$$

where $\Delta t = t_{j+1} - t_j$, $m = \exp(-\Delta t/\tau)$, and $\{w_j\}$ is a white noise sequence with variance $(1 - m^2) \sigma_{\epsilon}^2(\infty)^2$.

This model is not completely general, but it seems to adequately compensate for model errors in space navigation problems (Refs. 6 and 7). Accelerations which might otherwise be modeled as constant parameters are allowed to have modest time variations. By acknowledging such variations, the pitfall of processing data over a large period of time is avoided, thus bypassing the computation of acceleration error uncertainties that are unacceptably small.³ The inclusion of process noise prevents the filter from being restricted by the past behavior of the data. When process noise is not included, it often happens that the filter diverges when too much data is pro-

cessed. Using the **U-D** factored form of the Kalman filter, we express the filter mechanization in the following sequential steps⁴:

(1) State propagation

$$\tilde{\epsilon} = \Phi \hat{\epsilon}$$

$$\hat{\epsilon} = \epsilon(t_j | t_{j-1}), \text{ and } \tilde{\epsilon} = \epsilon(t_{j+1} | t_j)$$

(2) Deterministic covariance update

$$\tilde{\mathbf{P}} = \tilde{\mathbf{U}} \tilde{\mathbf{D}} \tilde{\mathbf{U}}^T = (\Phi \hat{\mathbf{U}}) \hat{\mathbf{D}} (\Phi \hat{\mathbf{U}})^T$$

(3) Process noise covariance update

$$\tilde{\mathbf{P}} = \tilde{\mathbf{U}} \tilde{\mathbf{D}} \tilde{\mathbf{U}}^T = \tilde{\mathbf{U}} \tilde{\mathbf{D}} \tilde{\mathbf{U}}^T + \text{Diag}(0, 0, \sigma_w^2)$$

$\tilde{\mathbf{U}}\tilde{\mathbf{D}}$ are obtained using the Agee-Turner rank one update algorithm of Ref. 5. $\hat{\mathbf{U}}\hat{\mathbf{D}}$ refers to filter values at time j , and $\tilde{\mathbf{U}}\tilde{\mathbf{D}}$ refers to the predicted values at time $j+1$.

(4) Preliminary measurement update calculations

$$\mathbf{f} = \tilde{\mathbf{U}}^T \mathbf{a}^T; \quad \mathbf{a}^T = (1, \Delta t, \Delta t^2/2)$$

$$\mathbf{g} = \tilde{\mathbf{D}} \mathbf{f}$$

$$\alpha = \sigma_{\Delta z}^2 + \mathbf{f}^T \mathbf{g} \text{ (innovations variance)}$$

(5) Residual validation

$$\Delta z = z - \mathbf{a}^T \tilde{\epsilon}$$

(6) Kalman gain computation

$$\mathbf{v} = \tilde{\mathbf{U}} \mathbf{g} \text{ (normalized gain)}$$

(7) Measurement covariance update

$$\hat{\mathbf{P}} = \hat{\mathbf{U}} \hat{\mathbf{D}} \hat{\mathbf{U}}^T = \tilde{\mathbf{U}} \left(\tilde{\mathbf{D}} - \frac{1}{\alpha} \mathbf{g} \mathbf{g}^T \right) \tilde{\mathbf{U}}^T$$

The **U-D** measurement update formulae of Ref. 3 simultaneously computes \mathbf{v} , $\hat{\mathbf{U}}$ and $\hat{\mathbf{D}}$; $\hat{\mathbf{U}}\hat{\mathbf{D}}$ represents $\mathbf{P}(t_{j+1} | t_{j+1})$

³Smaller than experience would predict or smaller than is consistent with the observed residuals.

⁴Time subscripts are omitted. It is assumed that we start with “~” filter quantities at step j , perform a time update, steps (1) – (3), and obtain “~” predict quantities at step $j+1$; and complete the cycle by computing “” filter quantities, steps (4) – (7), which refer to time $j+1$.

(8) State update

$$\hat{\epsilon} = \tilde{\epsilon} + v (\Delta z / \alpha)$$

$$\hat{\epsilon} \text{ now represents } \epsilon(t_{j+1} | t_{j+1})$$

Figure 2 illustrates one way in which this algorithm could be configured for the Tracking Real-Time Monitor Computer. Conditioning or pre-processing would be required to properly calibrate the incoming data. A start-up module could optimally be included to compute the *a priori* statistic and initial values required for an initial start. These can be computed from one or two passes of range and doppler data. The other two functions contain the Kalman sequential filter and its concomitant logic. Steps (1) – (5) would be performed in the first functional unit, while Steps (6) – (8) would be performed in the remaining functional unit.

Consideration could also be given to placing the algorithm at each station since all data and parameters required are available at each site. Figure 3 points out how this algorithm could be utilized within the existing radio metric data collection and processing system.

IV. Results

The filter algorithm just described was coded and tested on the Univac 1108 computer using simulated data. The data were produced assuming that the trajectory divergency phenomenon could be represented by a second-order polynomial. The data were perturbed by adding noise from a Gaussian pseudo-random number generator. Several tests using simulated data were executed with unvarying success. The following test parameters represent one of the more significant:

Modulo number = 100 μ s

Polynomial coefficients used to initialize the simulation estimate:

$$\epsilon_0 = 98 \mu\text{s} \approx 15\text{-km position error}$$

$$\dot{\epsilon}_0 = 10 \mu\text{s/day} \approx 35\text{-mm/s velocity error}$$

$$\ddot{\epsilon}_0 = 0.05 \mu\text{s/day}^2 \approx 2 \times 10^{-6} \text{-mm/s}^2 \text{ acceleration error}$$

Filter *a priori* uncertainties:

$$\sigma_{\epsilon} = 0.2 \mu\text{s} \approx 30 \text{ m}$$

$$\sigma_{\dot{\epsilon}} = 0.05 \mu\text{s/day} \approx 0.17 \text{ mm/s}$$

$$\sigma_{\ddot{\epsilon}} = 0.001 \mu\text{s/day}^2 \approx 4 \times 10^{-8} \text{ mm/s}^2$$

Simulation parameters:

$$\eta(t) = N(0, 100), 100 \text{ nanosecond (ns) gaussian white noise}$$

$$\Delta t_1 = 0.08\text{-day (2-h) sample interval used to generate data residuals}$$

$$\Delta t_2 = 0.3 \text{ day (7.2 h), length of data span}$$

$$\Delta t_3 = 20 \text{ days, time between data spans}$$

$$\Delta t_m = t_i - t_{i-1} = 1\text{-yr interval between state estimate time updates.}^5$$

$$\tau = 7 \text{ days}$$

To compress the results for illustration purposes, the filter residuals were averaged on a pass-by-pass basis (i.e., over each data span) over the one year period of the test. The upper graph in Fig. 4 represents the residual output of the filter, Δz , while the lower graph represents the innovations variance, α .

The filter reacted well to the simulated data; the residuals remained within bounds, and the innovations variance converged to the level of noise impressed on the data. This is a bit surprising because the filter model does not quite match the simulated data model, and the nature of the rollover effects is highly nonlinear. One rather interesting feature of this simulation that led us to pursue more conclusive testing was the observation that here was a case (although admittedly fabricated) where the range residuals could be tracked over an entire one-year period without the need of a trajectory update. Note that the space between tracking periods was 20 days.

In order to further verify the filter formulation, data from several spacecraft were collected and processed. The results from processing range data from the Viking A mission are presented here.

In processing these data, it was found that there was a greater sensitivity to the *a priori* model statistics and filter parameters than in the simulated cases. Although the divergence model matched the long term behavior of the observable, there were local trends in the data that tended to weaken

⁵Using this large time step corresponds to using a constant parameter model; the value of τ in this case is immaterial.

the predictive capability of the filter, thus preventing accurate processing over more than two or three days. The general nature of this data is illustrated in Fig. 5 with the scale distorted to illustrate the local features.

At this point the stochastic update feature of the filter became extremely important. After some experimentation, it was found that accurate estimates could be obtained with the evidently mismatched model if appropriate filter parameter values were chosen, viz., the epoch or state mapping interval, Δt_m , and the time constant or correlation interval, τ . The following set of test parameters permitted us to validly process 4 months of randomly spaced data with some gaps of 1-1/2 to 2 weeks:

$$\text{Modulo number} = 1000 \mu s$$

$$\epsilon_0 = 293 \mu s$$

$$\dot{\epsilon}_0 = 0.0004 \mu s/\text{day}$$

$$\ddot{\epsilon}_0 = 1 \mu s/\text{day}^2$$

$$\sigma_\epsilon = 1.0 \mu s$$

$$\sigma_{\dot{\epsilon}} = 1.0 \mu s/\text{day}$$

$$\sigma_{\ddot{\epsilon}} = 1.0 \mu s/\text{day}^2$$

$$\eta(t) = 50 \text{ ns}$$

$$\Delta t_m = 0.03 \text{ days (45 h)}$$

$$\tau = 14 \text{ days}$$

The adaptability of the filter to the data is demonstrated by the sample of the Viking A range data results. Figure 6 shows the filter results for 6 passes of data between 25 October and

4 December 1975. This data sequence began approximately 50 days after spacecraft launch and has data gaps of up to 18 days. The quality of the data fit is demonstrated by noting that in Fig. 5 the input data were on the order of microseconds (10^{-6} seconds), whereas the filter in this case has reduced the residuals to the nanosecond (10^{-9} second) level. The anomalous behavior of the data on 1 December 1975 which had not been previously detected in the original data is clearly in evidence. The anomalous behavior has not yet been verified because station logs are not detailed enough to reflect hardware malfunctions at this level. It is believed, however, that the anomaly is attributable to either hardware noise or a charged particle pulse in the transmission media. More analysis of this and other data phenomena remains. There is, however, no doubt that achieving the type of results would have been difficult if not impossible using the conventional least squares filter techniques that are commonly used for orbit determination.

V. Conclusions

The tests using both simulated and real data demonstrated that satisfactory real-time, automated estimation of our non-linear ranging process is achievable. The factorization filter algorithm design is compact and efficient, and is suitable for use in a mini-computer or micro-processor where processing time and memory constraints are important considerations.

Further tests must be performed before this ranging residual estimation technique can be determined suitable for use during actual operations. Future studies are planned to investigate model modifications to account for a diurnal velocity term that is often encountered in the actual data and for the inclusion of doppler data to improve the filter predictive characteristics. A final filter design will be arrived at by tuning the filter parameters so that acceptable performance is achieved under the various conditions that the filter is expected to operate.

References

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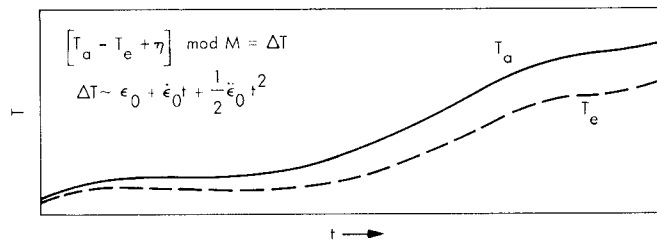


Fig. 1. Divergence of predicted line-of-sight range

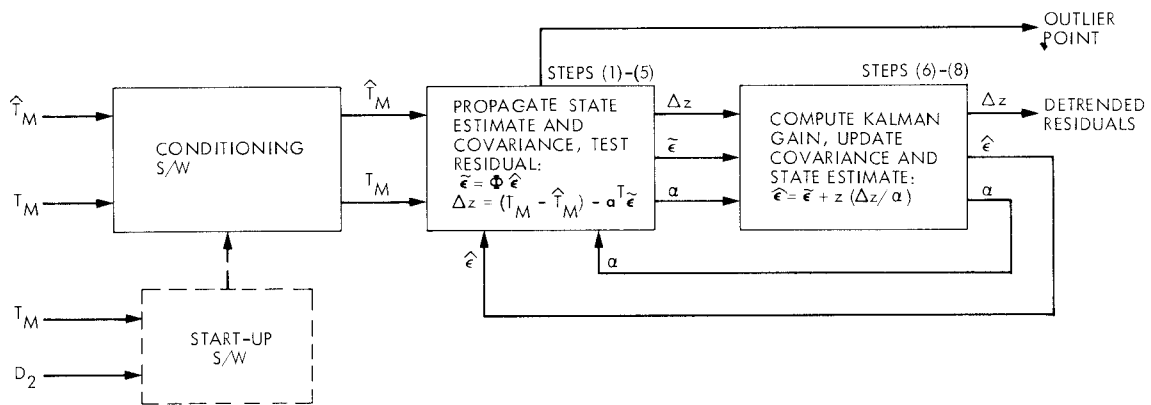


Fig. 2. Possible RTM mechanization of Kalman filter



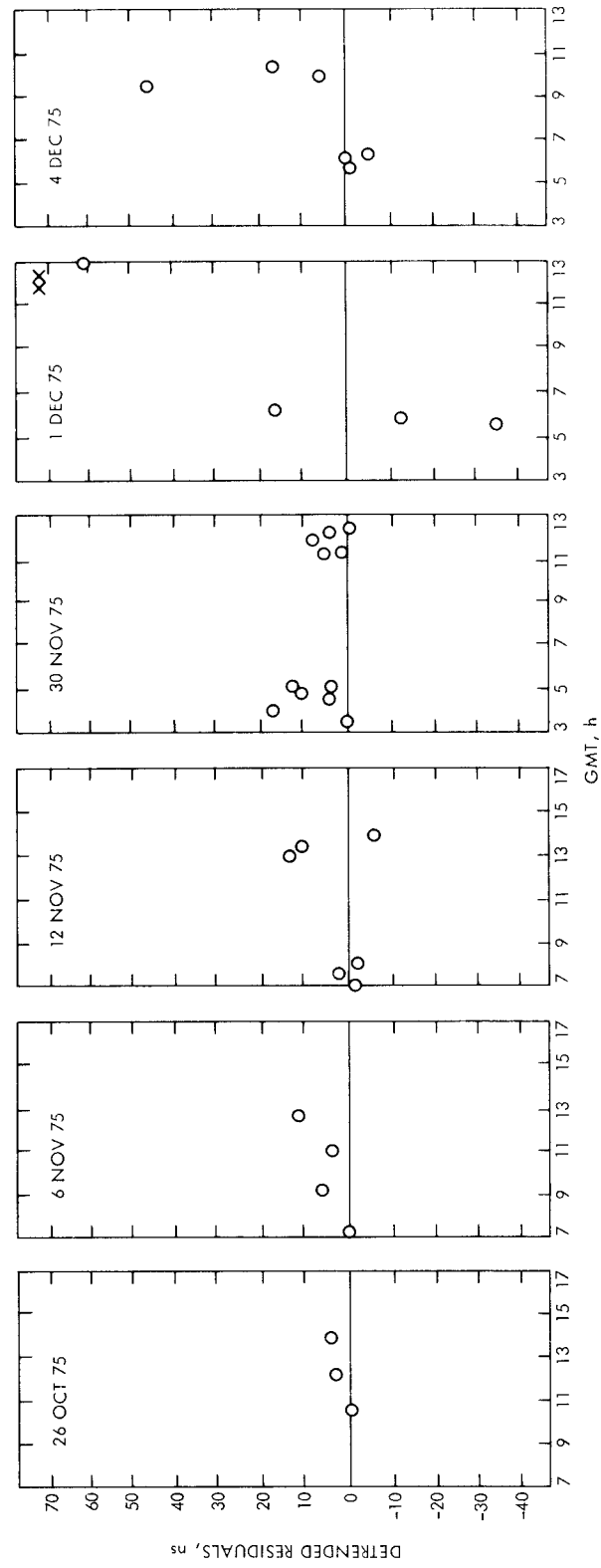


Fig. 6. Viking range residuals after Kalman filter detrending